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The Mathematical Implications of Lying

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Introduction. This article explores how statistics can be interpreted in different ways to yield different conclusions. It describes a pair of class activities. In the first, the results are interpreted to "show" that taking a group rather than an individual perspective is ultimately beneficial to the individual. In the second, a variation is added "showing" that telling the truth is better that lying.

The Lesson. Here is a scenario I presented to a 5^{th} grade math class: you and a friend are trapped in a burning building. The door is blocked and the only way out is through a window. The window is not large enough for both of you to go through at the same time. You have two options. You can try to cooperate with your partner to see that you both get out quickly, or you can fight with your partner to assure that you get out first. These choices yield four possible results: 1) you and your friend cooperate with each other, and you both get out with minor, first-degree burns; 2) you fight and your partner cooperates, leaving you without any burns and your partner with severe, third-degree burns; 3) you cooperate and your partner fights, with the opposite results; or 4) you both fight, leaving the both of you with moderate, second-degree burns. Which is the best choice?

Next I assigned number values to the different results. No burns were 0, minor burns were 1, moderate burns were 2 and severe burns were 3. During discussion I pointed out that higher numbers correspond with more severe burns, and the students agreed with me that the goal was to receive the smallest possible number. I asked students if, using this as a guide, it was better to fight or cooperate to get out of the burning building.

In the past I had difficulty getting the students in this class to cooperate with each other. Although there were exceptions, as a group these students were unusually competitive, so I was not surprised when the consensus of the class was that, based on the possible numerical outcomes, one would be better off fighting with their friend than cooperating. Since the range of outcomes for cooperators is 1 (minor burns) to 3 (severe burns), while for fighters the range is 0 (no burns) to 2 (moderate burns), it appears that fighting is the most reasonable choice.

Fight vs. Cooperate. We then played the game. I went around the class and randomly paired up the students. When it was a pair's turn to go, both students wrote on a piece of scrap paper either a "C," for cooperate, or an "F," for fight. Students did not know what their partners wrote. Then the two students showed the class what decision they made, each student was assigned a number based on the severity of burns they received. The decisions and scores were tallied on the board using the following format:

Group #	Decisions	Results
1	F-C	0-3
2	F-F	2-2
3	C-C	1-1
4	C-F	3-0
Etc.		

Each member of the class had an opportunity to participate, and we then took time to discuss and analyze the results. The first question I asked was, "How do we find out what was the average severity of burns received by the people in this class?" After a brief discussion we decided that the best way would be to add up all the numbers and divide by the number of students. I no longer have the exact results that we recorded that day, but there were 28 students in the class and a roughly equal number of C and F responses, so the final tally looked something like this:

Group #	Decisions	Results
1	F-C	0-3
2	F-F	2-2
3	C-C	1-1
4	C-F	3-0
5	C-C	1-1
6	C-F	3-0
7	F-C	0-3
8	C-F	3-0
9	F-F	2-2
10	C-C	1-1
11	F-C	0-3
12	F-F	2-2
13	C-C	1-1
14	C-F	3-0

I will use these numbers as I relate the discussion we had after we played the game.

Class Discussion. After performing the math and rounding to three digits, we determined that the average severity of burns received was 1.46--(forty-one total "burn" points received divided by 28 students participating). Since this was an average (the mean), the class agreed that, as this number went lower, an individual was less likely to receive severe burns and as it went higher, an individual was more likely to receive severe burns. The next step was to compare the results of the groups with cooperating students to the groups with fighting students. Our goal was to determine the likelihood, or probability, of burn severity for each type of decision pairing. We

made two columns, Cooperate and Fight, and tallied pair totals under each column. We had no difficulty assigning values when both pair members made the same decision. If they both cooperated the result was 1 + 1, so there were two points added to the Cooperate column, and if they both fought they received 2 + 2 points, so a 4 was added to the Fight column. We needed to make a decision when the pairs did not make the same choice. When the members of a pair made different decisions their score was 0 + 3, but we couldn't put a 3 under both columns because that would be adding their score twice, and that wouldn't give us an accurate average. Since we were tallying the results of each pair's decisions, I suggested that the best thing to do would be to split that pair's score in half and add the half score (1.5 points) to both columns. The students agreed to this solution.

We then totaled the score for each decision and divided the total by the number of people making that decision. Doing this using the numbers above yields the following results:

Group	Cooperate	Fight
1	1.5	1.5
2	-	4.0
3	2.0	-
4	1.5	1.5
5	2.0	-
6	1.5	1.5
7	1.5	1.5
8	1.5	1.5
9	-	4.0
10	2.0	-
11	1.5	1.5
12	-	4.0
13	2.0	-
14	1.5	1.5

The next step was to total the scores for each column and divide the total by the number of participants in each column. These totals were, for the Cooperate group, 18.5 divided by 15, or around 1.23, and for the Fight group, 22.5 divided by 13, or 1.73. This meant that, if you were in a cooperating group you were likely to have less severe burns. After seeing these totals, most students concluded that you were better off to be in a group that had at least one person cooperating.

Interpretation. It is important to realize that this result is only apparent when you look at the probable burn severity for the pair. If you restrict your evaluation to just an individual's burn severity, it appears that fighting to get out is a more advantageous reaction.

Yet when considering the overall probability for the entire group, it appears better to cooperate than to fight. The statistics show that it is preferable to be in a group where at least one person cooperated (29 points divided by 22 students in 11 groups = 1.32) than to be in a group where at least one person fought (34 points divided by 20 students in 10 groups = 1.70). However, to come to this conclusion you must ask the question, "Which decision is most likely to benefit the

most people?" In order to arrive at the conclusion reached above, the students needed to shift their perspective from a self-centered view to a group view.

One could argue that this exercise is mathematically misleading, because if you tallied individual scores the fighters would have a lower average than cooperators. (Cooperate: 29 points divided by 15 students = 1.93 average burn severity; Fight: 12 points divided by 13 students = 0.92 average burn severity.) But this is exactly the point of the exercise; the best decision depends on whether students consider only their own needs or if they think of themselves as members of a group. When taking a group perspective, a person is more likely to make decisions that benefit the entire group. In this game, as more people cooperate, the average gets lower and everyone becomes less likely to receive severe burns. As the entire group benefits, the individuals in the group also benefit.

Truthfulness vs. Lying. I tried this exercise a second time with a different class, and this time I decided to add another element to the game. Honesty was an issue with this particular group, and I was always looking for ways to impress upon them the value of honesty. We completed the exercise and discussed the results, and then I told the students that we would try it again, but with slightly different rules. Before a student wrote down a C or F to represent their decision, they would tell their partner (and the class) what they were going to write. The twist was that they didn't have to tell the truth; they could lie if they didn't want their partner to know what their decision was. We wrote down their intentions on the board so we could tell later if they were telling the truth or lying. After announcing what they would do, the students wrote down their decisions and showed them to the class. On the board, we kept track of the results of the second round this way:

Group #	Decisions	Results	Truth or Lie?
1	F-C	0-3	T-T
2	F-F	2-2	T-L
3	C-C	1-1	T-T
4	C-F	3-0	L-L
Etc.			

Interpretation. After everyone had an opportunity to go a second time we totaled the scores for the honest responses and compared them with the lies. We used a method similar to the one we used for the Fight or Cooperate groups: if both members of the group did the same thing we tallied the complete amount for the group, if they did different things we split their score in half. Once again, I don't have the original results, but these results are representative:

Group #	Decisions	Results	Truth or Lie?
1	F-C	0-3	T-T
2	F-F	2-2	T-L
3	C-C	1-1	T-T
4	C-F	3-0	L-L
5	C-C	1-1	T-T
6	C-F	3-0	T-T
7	F-C	0-3	L-T
8	C-F	3-0	T-T
9	F-F	2-2	L-L
10	C-C	1-1	T-T
11	F-C	0-3	L-T
12	F-F	2-2	L-L
13	C-C	1-1	T-T
14	C-F	3-0	L-L

In this example 17 students told the truth and 11 lied, which closely approximates the results that we achieved in class. Tallying the severity of burns for members of each group yields the following:

Group	Truth	Lie
1	3	-
2	2	2
3	2	-
4	-	3
5	2	-
6	3	-
7	1.5	1.5
8	3	-
9	-	4
10	2	-
11	1.5	1.5
12	-	4
13	2	-
14	-	3

The average severity of burns for the truth-tellers worked out to 22 points divided by 17 students, or 1.29, rounded off to three digits. For the liars, the average was 19 points divided by 11 students, or 1.90. These results were even more dramatic than the Fight vs. Cooperate results, and initiated a lively discussion. Next we checked the results for the different possible group outcomes (both truthful, both lying, or mixed) and got the following:

Group	Both Truthful	Both Lying	One Truthful, One Lying
1	3	-	-
2	-	-	4
3	2	-	-
4	-	3	-
5	2	-	-
6	3	-	-
7	-	-	3
8	3	-	-
9	-	4	-
10	2	-	-
11	-	-	3
12	-	4	-
13	2	-	-
14	-	3	-

Averaging burn severity for individuals in each group worked out to:
Both Truthful - 17 points divided by seven groups equals 2.43 points per group, or 1.21 per individual (the slight discrepancy is due to rounding)
Both Lying - 14 points divided by 4 groups equals 3.50 points per group, or 1.75 per individual
One Truthful, One Lying - 10 points divided by three groups equals 3.33 points per groups or 1.67 per individual.

Class Discussion. Clearly, a student that lied was more likely to receive severe burns in this game. When asked to explain why this happened, the students were quick to notice that they were more likely to lie if they were planning to fight their partner. (Of the 11 students who lied, 9 chose to fight and only 2 chose to cooperate.) When asked to draw conclusions from this exercise, one conclusion arrived at was that people are more likely to lie if they are planning to do something bad to someone. Another comment was that people only lie if they are not proud of what they are planning to do.

Conclusion. Honesty and cooperation were still issues in these classrooms after the lesson, although as the year progressed the situation improved. It is difficult to assess what impact, if any, this exercise had on the students' behavior toward each other. There were many discussions about honesty and cooperation in these classrooms. Still, I think that having it displayed, however imperfectly, as part of a math lesson made a greater impression on the students than just telling them to cooperate with each other and to always be truthful.

Extension: In the example as it is, "fight" always produces the same result. A reviewer pointed out that the exercise could be made more realistic, though more complex, by putting probabilities on the outcomes when fights occur, so there would be some probability that a fighter would win.

Glenn Simonelli has taught elementary school in Bloomington, Indiana for six years. He is currently on a leave of absence to complete a doctorate degree in science education.